Heisenberg-limited sensitivity without entanglement

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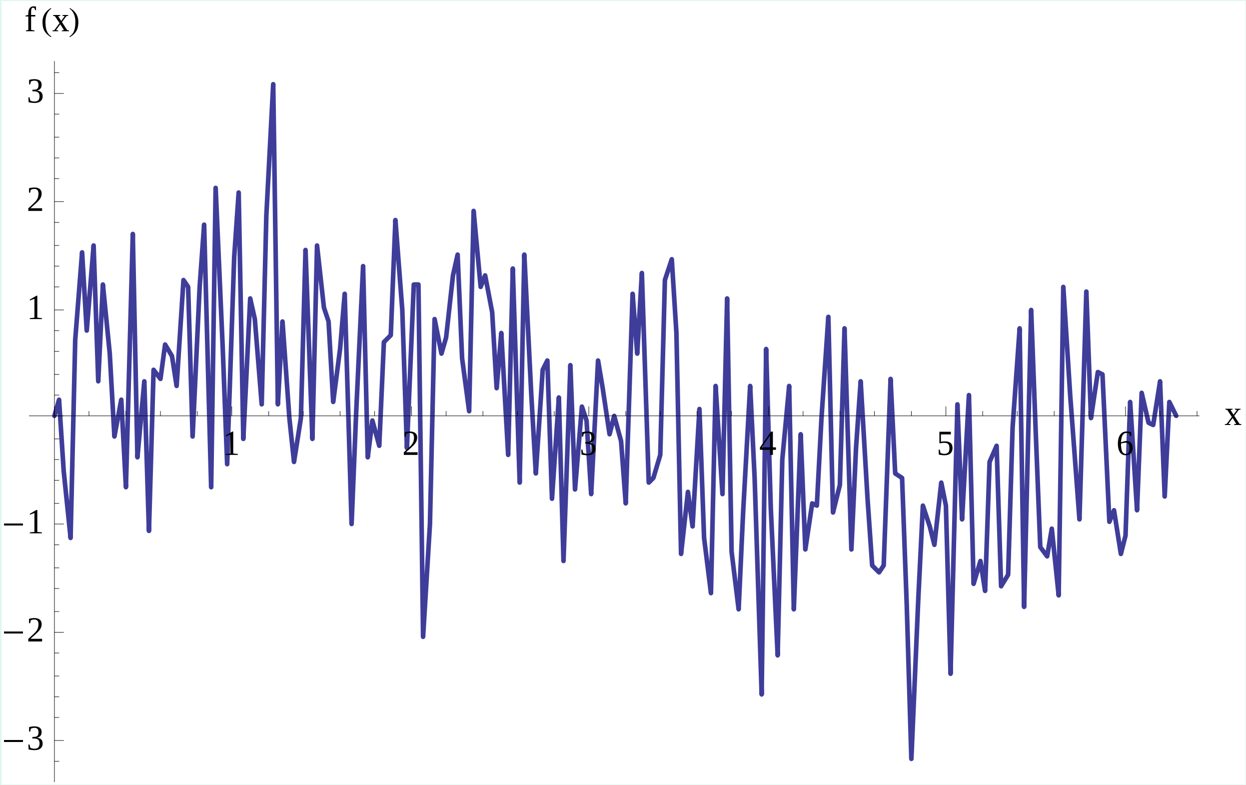
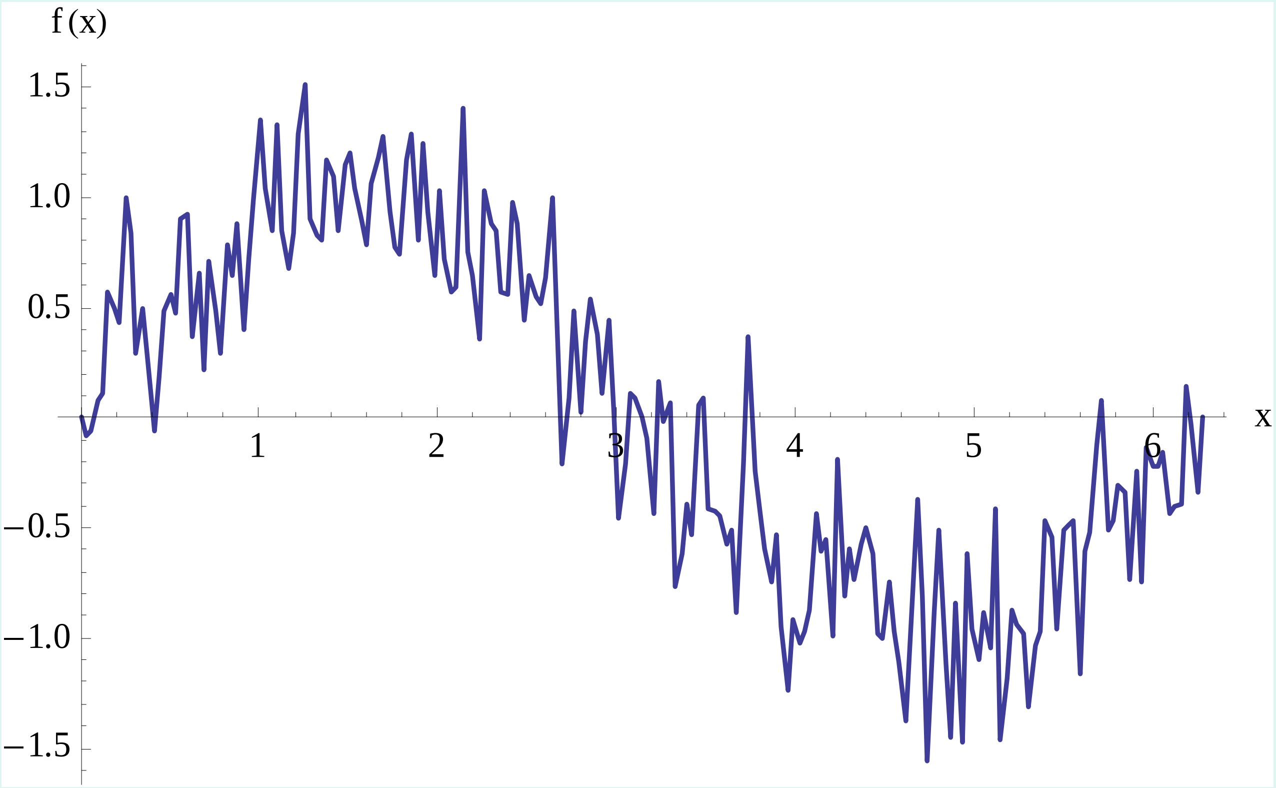
Improving S/N



N=1

realization

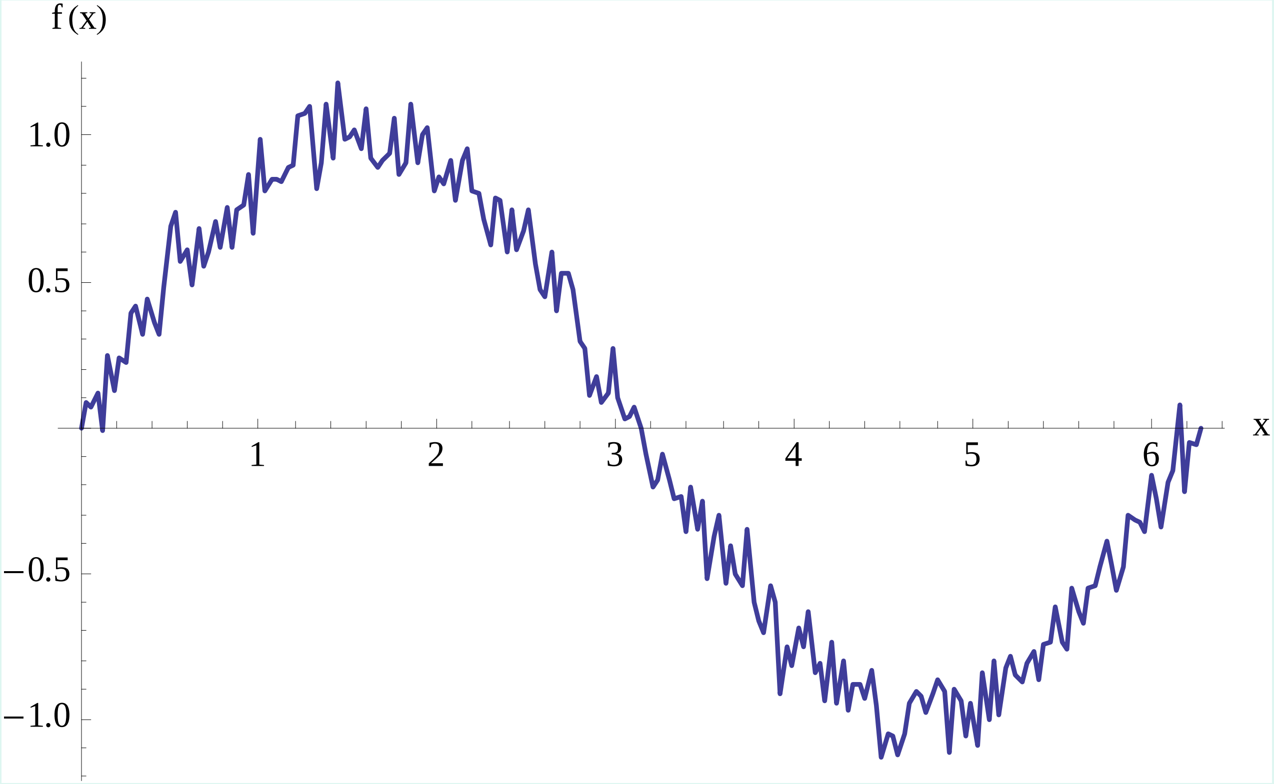
Improving S/N: averaging



N=10

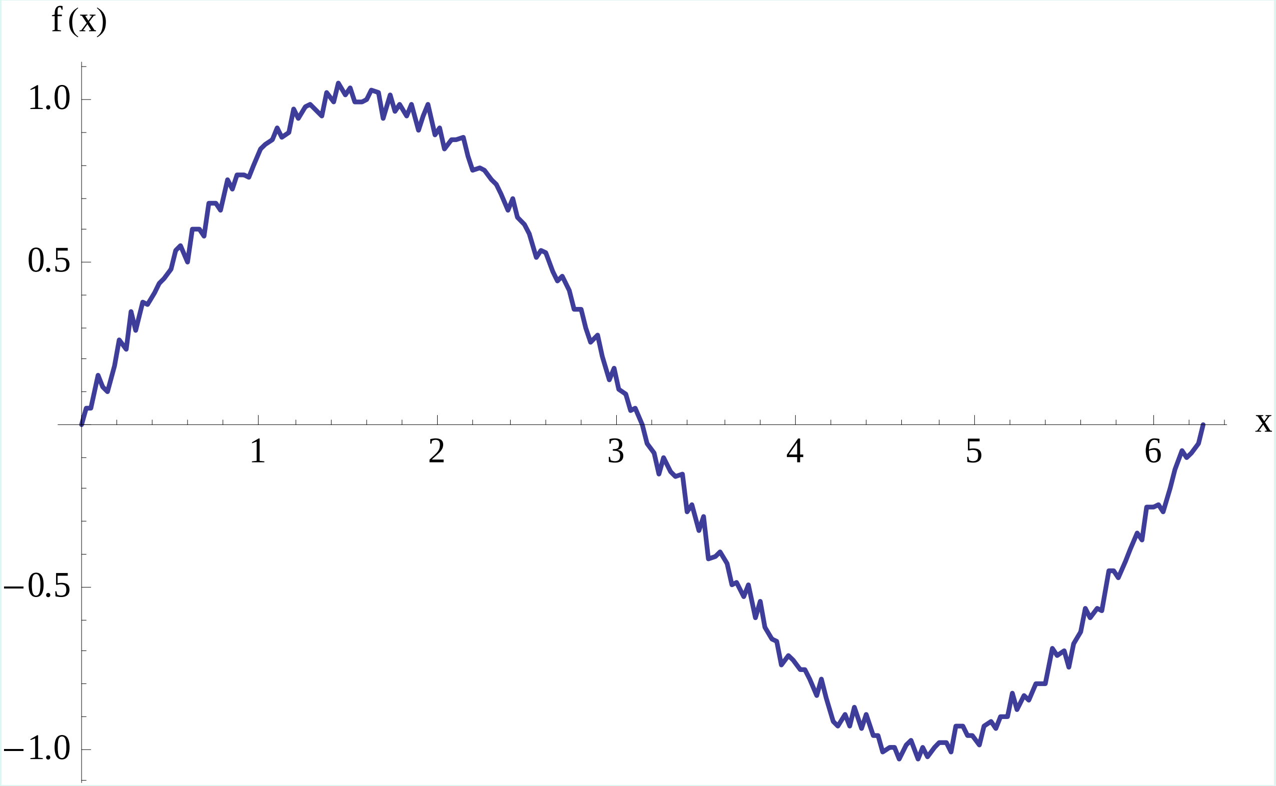
realizations

Improving S/N: averaging



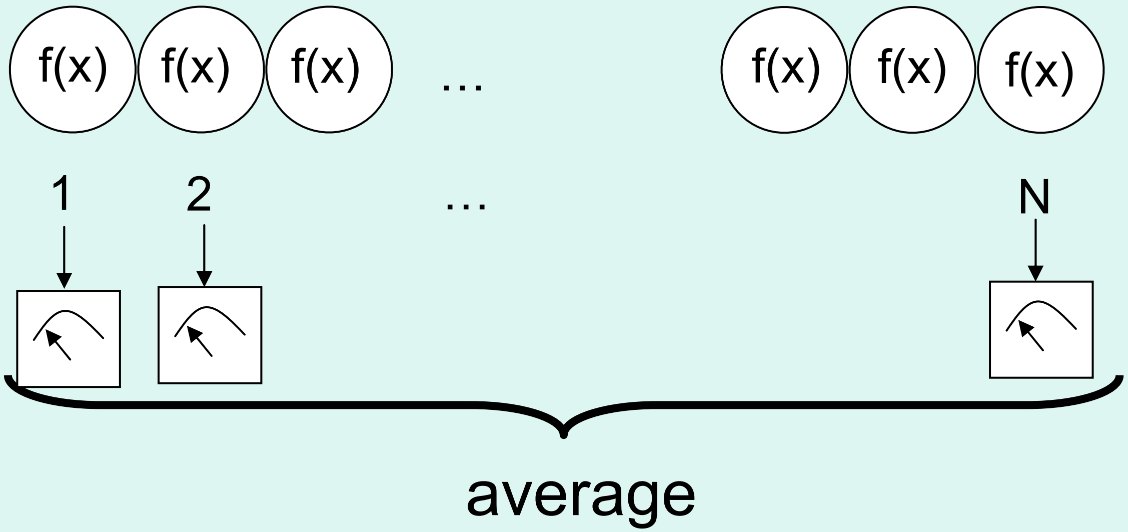
N=100
realizations

Improving S/N: averaging



N=1000 realizations

N independent, identically prepared systems



$$|\psi(0)\rangle = \otimes_{i=1}^{N} |\varphi\rangle_i$$

$$H(x) = \sum_{i=1}^{N} h_i(x)$$

Signal/Noise $\sim \sqrt{N}$

Best sensitivity $\delta x_{\min} \sim 1/\sqrt{N}$

Giovannetti et al. PRL ’06

“Standard Quantum Limit (SQL)”
“Shot noise limit”

N independent, identically prepared

$|\psi(0)\rangle = \otimes_{i=1}^{N} |\varphi\rangle_i$

$H(x) = \sum_{i=1}^{N} h_i(x)$

Signal/No…

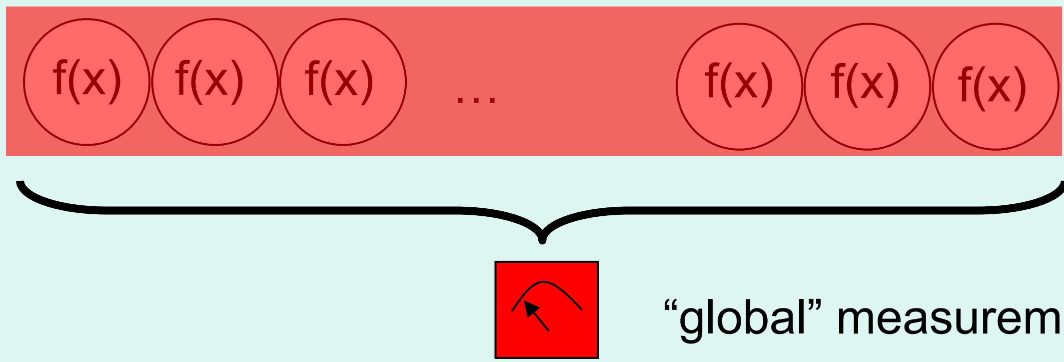
wikipedia

Abraham de Moivre 1733
Central limit theorem

Best sens…

“Standard Quantum Limit (SQL)”
“Shot noise limit”

Quantum Enhanced Measurements: *N* entangled systems



"global" measurement

$$|\psi(0)\rangle \neq \otimes_{i=1}^{N} |\varphi\rangle_i$$

$$H(x) = \sum_{i=1}^{N} h_i(x)$$

Signal/Noise $\sim N$

Best sensitivity $\delta x_{\min} \sim 1/N$

[Hollenhorst PRD'79; Caves PRD '81; Dowling et al. PRA '98,PRL '00; Lloyd et al. Nature '01, Science '04, PRL '05, PRL '06, Nature Physics '11]

"Heisenberg limit"

Letters to Nature

Nature **412**, 417-419 (26 July 2001) | doi:10.1038/35086525; Receive

Quantum-enhanced positioning and clock synchronization

Vitto... ...annetti[1], Seth Lloyd[2] & Lorenzo Maccone[1]

News and Views

Nature **457**, 35-36 (1 January 2009) | doi:10.1038/457035a; Publis

Quantum physics: Squeeze until it hurts

Geoff J. Pryde[1]

Atom-chip-based generation of entanglement for quantum metrology

Max F. Riedel, Pascal Böhi, Yun Li, Theodor W. Hänsch, Alice Sinatra & Philipp Treutlein

Affiliations | Corresponding authors

Nature **464**, 1170–1173 (22 April 2010) | doi:10.

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‹ Prev | Tabl

Science 12 September 2008:
Vol. 321 no. 5895 pp. 1463-1465
DOI: 10.1126/science.1160627

REPORT

Enha...ed Sensitivity of Photodetection via Quantum Illumination

Seth

Nature **429**, 161-164 (13 May 2004) | doi:10.1038/nature02493; Rece
Accepted 16 March 2004

Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg

Nature **443**, 316-319 (21 September 2006) | do
Accepted 18 July 2006

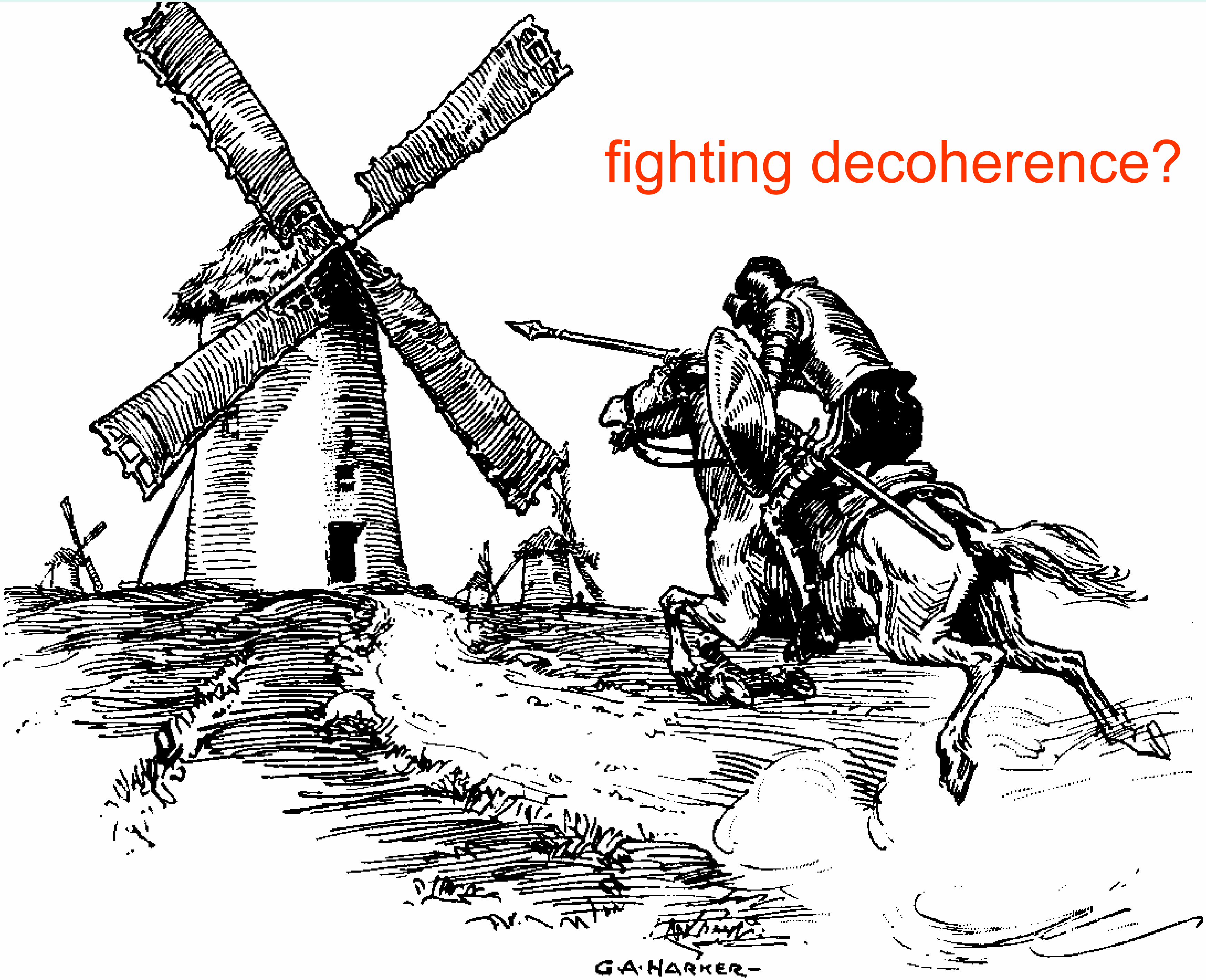
'Designer atoms' for quantum metrology

C. F. Roos[1,2], M. Chwalla[1], K. Kim[1], M. Riebe[1] & R. Blatt[1,2]

BUT: The slightest amount of decoherence leads back to $1/\sqrt{N}$ scaling (i.e. SQL) for large N!

S.Huelga et al. PRL '97
J.Kołodyński et al. PRA '10
L.Davidovich et al. Nature Physics '11

fighting decoherence?



This talk:

- Decoherence is the signal!

$\rightarrow$ HL with an initial product state

- How is it possible?

$\rightarrow$ Role of a “quantum bus”

- A general scheme for HL with initial product state

Decoherence free subspace (DFS)

$$\begin{aligned} \dot{\rho} &= \mathcal{L}_{\mathrm{sys}}[\rho] + \mathcal{L}_{\mathrm{D}}[\rho] \\ \mathcal{L}_{\mathrm{D}}[\rho] &= \sum_{\alpha} a_{\alpha} \left([F_{\alpha}, \rho F_{\alpha}^{\dagger}] + [F_{\alpha}\rho, F_{\alpha}^{\dagger}]\right) \end{aligned}$$

Lidar, Chuang, Whaley PRL '98

Condition for DFS states $|\gamma\rangle$

$$\mathcal{L}_{\mathrm{D}}[|\gamma\rangle\langle\gamma|] = 0 \;\Leftrightarrow F_{\alpha}|\gamma\rangle = 0 \quad \forall\alpha$$

(for $F_{\alpha} \in$ semi-simple Lie algebra)

DFS: W. Zurek '81; Rotational Tunneling '83-'90s;
QC: Duan & Guo '97, Zanardi '97,
Superradiance: DB, F.Haake, P.Braun '98,

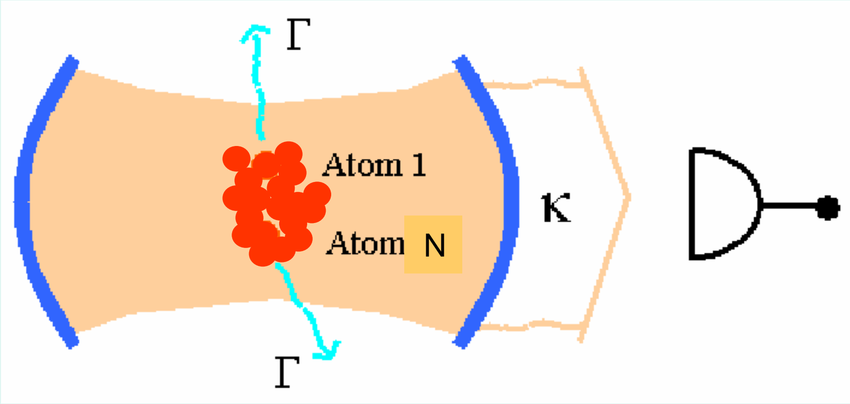
Example: Superradiance

$$\dot{\rho} = \mathcal{L}_c[\rho] \equiv \gamma\left([J_-\rho, J_+] + ([J_-, \rho J_+]\right), \quad \gamma = \frac{g^2}{\kappa}$$

$$J_- = \sum_{i=1}^{N} \sigma_-^{(i)}$$

$$\Gamma \ll g\sqrt{N} \ll \kappa$$
$$k_B T \ll \hbar\omega_0$$

Bonifacio, Schwendimann, Haake '71



- Collective emission of light
- Short bright pulse of light – duration ~1/N
- Decoherence time typically $\sim 1/N^2$ (|jm> basis)

DFS in superradiance

$$\text{DFS: } J_{-}|\psi\rangle = 0 \Rightarrow \ |\psi\rangle = |J \ -J\rangle_{JM}$$

- *J=0,…,N/2* (*N* even)
- High degeneracy
- There are $\binom{N}{N/2} \sim 2^N/\sqrt{N}$ exact DFS states

Beige, DB, Knight, NJP ‘00

Basic idea of “Decoherence-Enhanced Measurements” (DEM)

- DFS evolves when couplings change
- Protected superpositions become exposed to decoherence
- Proximity of states with extremely fast decoherence ($N \gg 1$) and DF states allows for very sensitive measurement of change of couplings

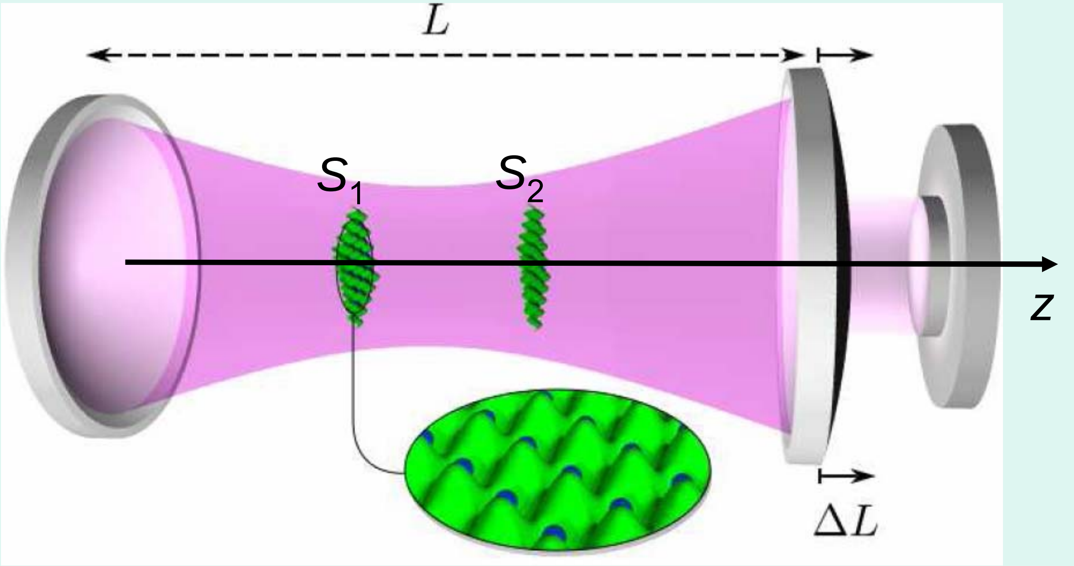
Measuring the length of a cavity

- 2 sets of atoms

$$S_1 \equiv \{1, \dots, N/2\}$$

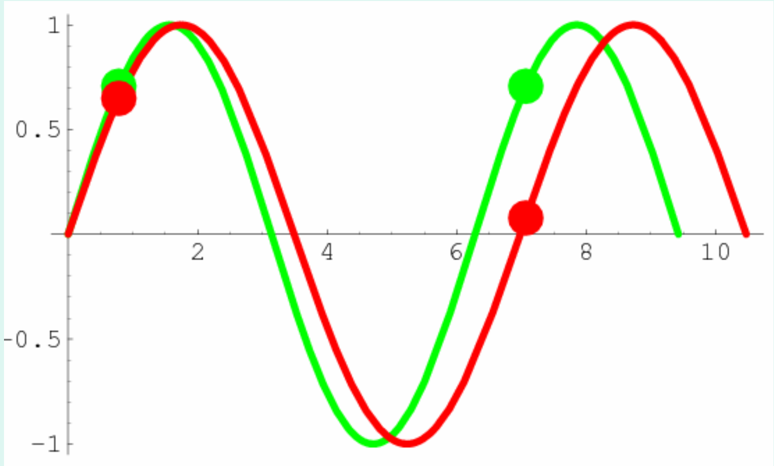
$$S_2 \equiv \{\tfrac{N}{2}+1, \dots, N\}$$

- Resonant couplings G_i for all atoms in set i



$$G_i = \sqrt{\frac{\hbar\omega}{\epsilon_0 V}} \sin(k_z z_i)\, \boldsymbol{\epsilon} \cdot \mathbf{d}$$

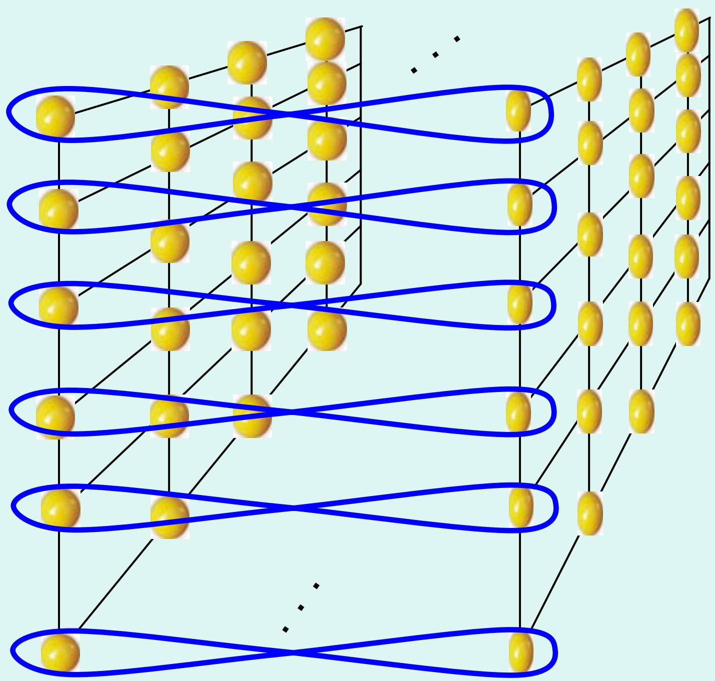
$$V = AL,\, k_z = n_z \pi / L$$



$$\text{change of } L \Rightarrow \text{collective change of } G_1, G_2$$

$$J_-(x) = J_-^{(s)} + xJ_-^{(a)}, \quad x \propto \delta L/L$$
$$J_-^{(s)} = J_-^{(1)} + J_-^{(2)}, \quad J_-^{(a)} = J_-^{(1)} - J_-^{(2)}$$

Initial product state of *pairs* of atoms *in DFS* *(dark states)*



$$|\psi_0\rangle = \bigotimes_{l=1}^{N/2} |\varphi\rangle_l$$

$$|\varphi\rangle_l = (\underbrace{|t_-\rangle_l}_{|00\rangle} + \underbrace{|s\rangle_l}_{(|01\rangle - |10\rangle)/\sqrt{2}})/\sqrt{2}$$

Photon statistics

$$\langle a^{\dagger m} a^m(t) \rangle = 2m \left(\frac{g}{\kappa}\right)^{2m} \int_0^t ds \kappa e^{-2m\kappa s} (e^{\kappa s} - 1)^{2m-1} \langle J_+^m J_-^m (t-s) \rangle$$

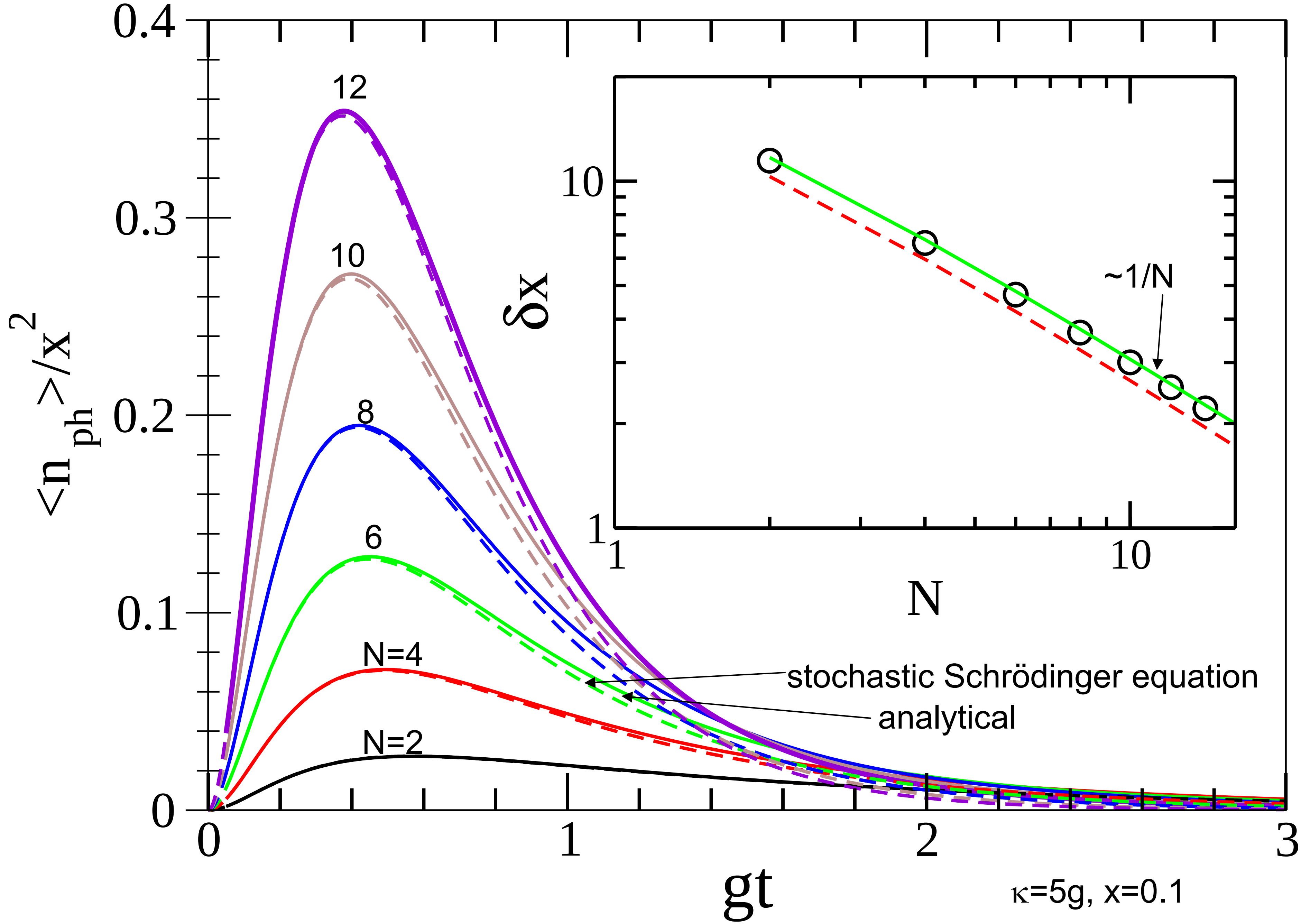
Bonifacio, Schwendimann, Haake '71

$$\langle n_{\rm ph}(x,t) \rangle = \frac{1}{2} g^2 t^2 x^2 \left(\frac{N}{2} (\frac{N}{2} + 1) \right) + \mathcal{O}(t^3)$$

regime of $\langle n_{ph} \rangle \ll 1$:

$$\delta x = \frac{\langle \Delta n_{\rm ph}^2 (x,t) \rangle^{1/2}}{\sqrt{M} |\frac{\partial}{\partial x} \langle n_{\rm ph}(x,t) \rangle|} = \frac{\sqrt{2}}{\sqrt{M} g t \sqrt{N(N+2)}}$$

HL with initial product state!



0.4
0.3
0.2
0.1
0
$<n_{ph}>/x^2$
12
10
8
6
N=4
N=2
0
1
2
3
gt
κ=5g, x=0.1
stochastic Schrödinger equation
analytical
10
1
δx
~1/N
1
10
N

How is it possible?

"quantum bus" $\mathcal{R}$

$$H(x) = \underbrace{\sum_{i=1}^{N} H_i + H_R}_{H_0} + \underbrace{\sum_{i,\nu} S_{i,\nu}(x) \otimes R_\nu}_{H_I(x)}$$

Quantum Parameter Estimation Theory

$$\delta x \geq \delta x_{\min} = \frac{1}{2\sqrt{M}\left(\frac{ds^2_{\mathrm{Bures}}}{dx^2}\right)^{1/2}}$$

Braunstein & Caves, '94

Quantum Cramér-Rao bound

number of measurements

If $$|\psi(x+dx)\rangle = \exp(-i\hat{h}dx)|\psi(x)\rangle$$

$$\Rightarrow \qquad ds_{\mathrm{Bures}} = \langle \Delta \hat{h}^2 \rangle^{1/2} dx$$

Simplest case: just interaction

$$H(x) = H_I(x) = x \sum_i S_i \otimes R$$

$$\Rightarrow \hat{h} = \sum_i S_i \otimes Rt$$

Initial pure product state of all subsystems; $S_i = S \; \forall i$

$$\Rightarrow \langle \Delta \hat{h}^2 \rangle = \left(N \langle \Delta S^2 \rangle \langle R^2 \rangle + N^2 \langle S \rangle^2 \langle \Delta R^2 \rangle \right) t^2$$

$$\delta x_{\min} \underset{N \gg 1}{=} \frac{1}{2\sqrt{M} t |\langle S \rangle| \langle \Delta R^2 \rangle^{1/2}} \frac{1}{N}$$

HL with initial product state!

- No contradiction to standard result: subsystems are *not independent*:

Giovannetti et al PRL ‘06

$$H(x) \neq x \sum_i h_i$$

Coupling to common quantum bus $\mathcal{R}$!

- SQL for *R=I* (= absence of quantum bus)
- Quantum fluctuations of *R* are important to achieve HL!

Measure what?

$$|\psi_0\rangle = \otimes_{i=1}^{N} |\varphi\rangle_i \otimes |\xi\rangle, \quad H_0 = 0 \qquad H_I(x) = \sum_i x S_i \otimes R$$

$$|\varphi\rangle_i = |s\rangle_i,\ S_i|s\rangle_i = s|s\rangle_i,\ |\xi\rangle = \sum\nolimits_m d_m|r_m\rangle \text{ with } R|r_m\rangle = r_m|r_m\rangle$$

$$|\psi(t)\rangle = \sum_m d_m e^{-ixNsr_m t} \otimes_{i=1}^{N} |s_i\rangle \otimes |r_m\rangle$$

- Collective phase between states of q-bus
- Revealed by any observable *A* on q-bus with [*A*,*R*]≠0!

$$A = |r_0\rangle\langle r_1| + |r_1\rangle\langle r_0|$$

General case

$$H(x) = \sum_{i=1}^{N} H_i + H_R + \underbrace{\sum_{i,\nu} S_{i,\nu}(x) \otimes R_\nu}_{H_I(x)}$$

- Initial product state $|\psi_0\rangle = \otimes_{i=1}^{N} |\varphi\rangle_i \otimes |\xi\rangle$
- Assume single particle dynamics and q-bus dynamics solved
- Treat $H_I(x)$ in perturbation theory up to 2nd order

q-PET

$$
\begin{aligned}
d^2_{\mathrm{Bures}}(\rho(x),\rho(x+dx)) &= dx^2\int_0^t\int_0^t dt_1 dt_2 K(x,t_1,t_2)\\
K(x,t_1,t_2) &= \langle H_I'(x,t_1)H_I'(x,t_2)\rangle - \langle H_I'(x,t_1)\rangle\langle H_I'(x,t_2)\rangle\\
&= \sum_{\nu,\mu}\Big(NC_{S'\nu\mu}(x,t_1,t_2)\langle R_\nu(t_1)R_\mu(t_2)\rangle\\
&\quad + N^2\langle S_\nu'(x,t_1)\rangle\langle S_\mu'(x,t_2)\rangle C_{R\nu\mu}(t_1,t_2)\Big)\\
C_{S'\nu\mu}(t_1,t_2) &= \langle S_\nu'(x,t_1)S_\mu'(x,t_2)\rangle - \langle S_\nu'(x,t_1)\rangle\langle S_\mu'(x,t_2)\rangle\\
C_{R\nu\mu}(t_1,t_2) &= \langle R_\nu(t_1)R_\mu(t_2)\rangle - \langle R_\nu(t_1)\rangle\langle R_\mu(t_2)\rangle
\end{aligned}
$$

- All operators in interaction picture w.r.t free hamiltonian H_0
- Variances replaced by time integrals of correlation functions
- Scaling with N^2 of K gives HL

Measuring the q-bus

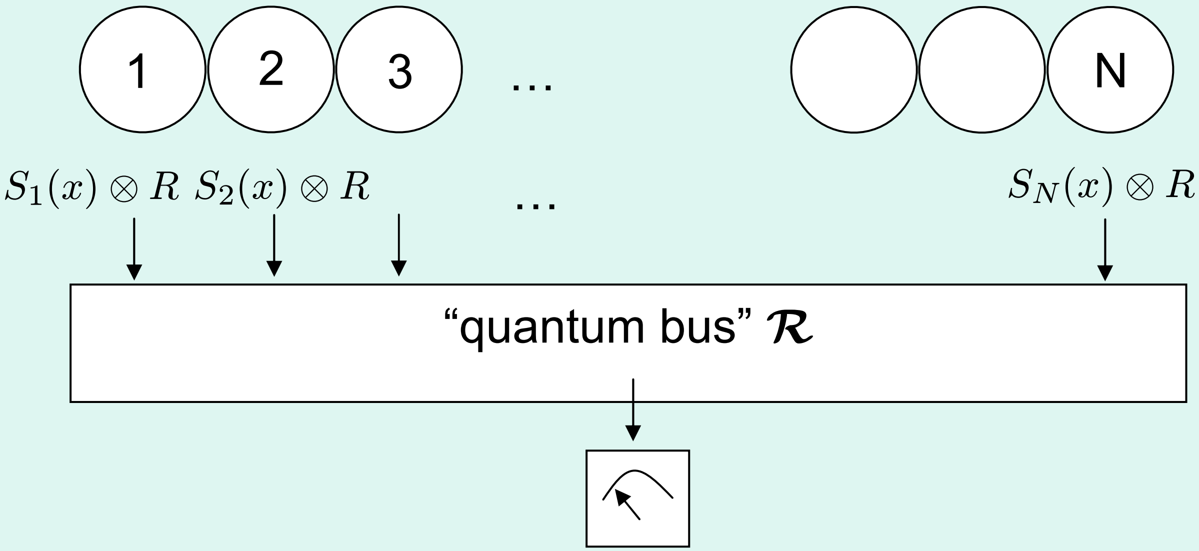
$$\delta x = \frac{\left[\int_0^t dt_1 \int_0^t dt_2 \sum_{\nu,\mu} \langle S_\nu(x,t_1)\rangle \langle S_\mu(x,t_2)\rangle \langle [R_\nu(t_1), A][A, R_\mu(t_2)]\rangle\right]^{1/2}}{\sqrt{MN}\left|\int_0^t dt_1 \int_0^t dt_2 \sum_{\nu,\mu} \frac{\partial}{\partial x}\left(\langle S_\nu(x,t_1)\rangle \langle S_\mu(x,t_2)\rangle\right) \langle R_\nu(t_1)[A, R_\mu(t_2)]\rangle\right|}$$

Simplifying assumptions:

$$A|\xi\rangle = a_\xi|\xi\rangle$$
$$[A, H_R] = 0$$

q-bus is in initial noiseless state concerning observable A and remains noiseless without interaction

N identically prepared systems interacting with common quantum bus



$$|\psi(0)\rangle = \otimes_{i=1}^{N} |\varphi\rangle_i$$

$$H_I(x) = \sum_{i=1}^{N} S_i(x) \otimes R$$

Signal/Noise $\sim N$

Best sensitivity $\delta x_{\min} \sim 1/N$

Heisenberg limit with initial product state

DB, J. Martin,
Nature Com. 2, 223 (2011)

Individual decoherence

- Markovian decoherence of all subsystems on top of unitary evolution

$$\dot{W}(t) = -\left(L_0 + L_I(x) + \Lambda_s + \Lambda_R\right) W(t)$$

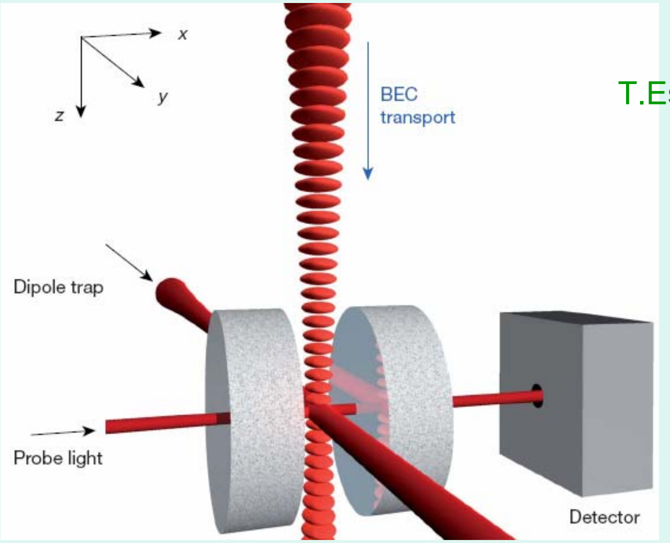
$$L_0\, X = [H_0, X],\ \ L_I(x)\, X = [H_I(x), X],\ \ \Lambda_s = \sum_{i=1}^{N} \Lambda_i$$

- Free evolution (*H_I=0*) still factorizes
- Expectation values/correlation functions w.r.t mixed states, but

$\Rightarrow$ Still 1/N scaling of δx

A long way to decoherence-enhanced LIGOs…

length measurement with sensitivity ~$10^{-21}/\sqrt{Hz}$ (2009)



T.Esslinger et al. Nature '07

- 3D optical lattice in cavity?
- diamonds with NV centers in cavities?

- 10^{21} atoms, HL $\Rightarrow \delta L/L \sim 10^{-21}$
- 0.1 μm atom spacing $\Rightarrow$ 1m^3 active volume…

Conclusions

- Heisenberg limit with initial product state
  - N systems coupled to common q-bus
  - Measure almost any observable of q-bus
  - Scaling robust against individual decoherence
- Decoherence can be the signal

arXive:0902.1213
arXive:1005.4443

Nature Communications **2**, 223 (2011)